1. A three-variable linear programming problem in x, y and z is to be solved. The objective is to maximise the profit P. The following initial tableau was obtained.

Basic variable	x	y	z	r	s	Value
r	2	0	4	1	0	80
s	1	4	2	0	1	160
P	–2	–8	–20	0	0	0

(a) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the simplex algorithm, to obtain tableau T. State the row operations that you use.

You may not need to use all of these tableaux.

Basic variable	x	y	z	r	s	Value	Row operations

Basic variable	x	y	z	r	s	Value	Row operations

Basic variable	x	y	z	r	s	Value	Row operations

(5)

(b) Write down the profit equation shown in tableau T.

(1)

(c) State whether tableau T is optimal. Give a reason for your answer.

(1)
(Total 7 marks)

2. The tableau below is the initial tableau for a linear programming problem in x, y and z. The objective is to maximise the profit, P.

Basic Variable	x	y	z	r	s	T	Value
r	0	1	2	1	0	0	24
s	2	1	4	0	1	0	28
t	–1	$\frac{1}{2}$	3	0	0	1	22
P	–1	–2	–6	0	0	0	0

(a) Write down the profit equation represented in the initial tableau.

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

(1)

(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.

(9)

(c) State the final value of the objective function and of each variable.

(3)
(Total 13 marks)

3. While solving a maximising linear programming problem, the following tableau was obtained.

Basic Variable	x	y	z	r	s	t	value
z	$\frac{1}{4}$	$-\frac{1}{4}$	1	$\frac{1}{4}$	0	0	2
s	$\frac{5}{4}$	$\frac{7}{4}$	0	$-\frac{3}{4}$	1	0	4
t	3	$\frac{5}{2}$	0	$-\frac{1}{2}$	0	1	2
P	–2	–4	0	$\frac{5}{4}$	0	0	10

(a) Write down the values of x, y and z as indicated by this tableau.

(2)

(b) Write down the profit equation from the tableau.

(2)
(Total 4 marks)

4. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays 1	*B* plays 2	*B* plays 3
A plays 1	5	7	2
A plays 2	3	8	4
A plays 3	6	4	9

(a) Formulate the game as a linear programming problem for player *A*, writing the constraints as equalities and clearly defining your variables.

(5)

(b) Explain why it is necessary to use the simplex algorithm to solve this game theory problem.

(1)

(c) Write down an initial simplex tableau making your variables clear.

(2)

(d) Perform two complete iterations of the simplex algorithm, indicating your pivots and stating the row operations that you use.

(8)
(Total 16 marks)

1. (a)

b.v	x	y	z	r	s	Value	Raw ops		
z	$\frac{1}{2}$	0	1	$\frac{1}{4}$	0	20	$R_1 \div 4$	M1 A1	
s	0	4	0	$-\frac{1}{2}$	1	120	$R_2 - 2R_1$	M1 A1 ft	
P	8	–8	0	5	0	400	$R_3 + 20R_1$	A1 ft	5

(b) $P + 8x - 8y + 5r = 400$ B1ft 1

(c) Not optimal since there is a negative number in the profit row B1ft 1

[7]

2. (a) $P - x - 2y - 6z = 0$ B1 1

Note

1B1: cao

(b)

b.v	x	y	z	r	s	t	Value
r	0	1	2	1	0	0	24
s	2	1	4	0	1	0	28
t	–1	$\frac{1}{2}$	3	0	0	1	22
P	–1	–2	–6	0	0	0	0

b.v.	x	y	z	r	s	t	Value	Row Ops.		
r	–1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$	M1 A1	
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 4$	M1 A1ft	
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	1	$R_3 - 3R_2$	A1	
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 6R_2$		5

b.v.	x	y	z	r	s	t	Value	Row Ops.		
y	–2	1	0	2	–1	0	20	$R_1 \div \frac{1}{2}$	M1 A1ft	
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$	M1 A1	
t	–3	0	0	$\frac{1}{2}$	–1	1	6	$R_3 + \frac{1}{4}R_1$		
P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$		4

Notes

1M1: correct pivot located, attempt to divide row
1A1: pivot row correct including change of b.v.
2M1: (ft) Correct row operations used at least once or stated correctly.
1A1ft: Looking at non zero-and-one columns, one column ft correct
2A1: cao.
3M1: (ft) Correct pivot identified – negative pivot gets M0 M0
1A1: ft pivot row correct including change of bv – but don't penalise b.v. twice.
4M1: (ft) Correct row operations used at least once or stated correctly.
1A1: cao

Misread Alternative 1

Increasing *x* first,

b.v.	x	y	z	r	s	t	Value	row ops
r	0	1	2	1	0	0	24	R_1 no change
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	14	$R_2 \div 2$
t	0	1	5	1	$\frac{1}{2}$	1	36	$R_3 + R_2$
P	0	$-\frac{3}{2}$	–4	0	$\frac{1}{2}$	0	14	$R_4 + R_2$

then *y* next,

b.v.	x	y	z	r	s	t	Value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
x	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{2}R_1$
t	0	0	3	–1	$\frac{1}{2}$	1	12	$R_3 - R_1$
P	0	0	–1	$\frac{3}{2}$	$\frac{1}{2}$	1	50	$R_4 + \frac{3}{2}R_1$

then *z*.

b.v.	x	y	z	r	s	t	Value	row ops
y	–2	1	0	2	–1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	–3	0	0	$\frac{1}{2}$	–1	1	6	$R_3 - 3R_2$
P	0	0	0	1	1	1	52	$R_4 + R_2$

Misread Alternative 2

Increasing *x* first,

b.v.	x	y	z	r	s	t	Value	row ops

r	0	1	2	1	0	0	24	R_1 no change
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	14	$R_2 \div 2$
t	0	1	5	0	$\frac{1}{2}$	1	36	$R_3 + R_2$
P	0	$-\frac{3}{2}$	–4	0	$\frac{1}{2}$	0	14	$R_4 + R_2$

Increasing *z* next,

b.v.	x	y	z	r	s	t	Value	row ops
r	–1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 2$
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	1	1	$R_3 - 5R_2$
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 4R_2$

then increasing *y*.

b.v.	x	y	z	r	s	t	Value	row ops
y	–2	1	0	2	–1	0	20	$R_1 \div \frac{1}{2}$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$
t	–3	0	0	$\frac{1}{2}$	–1	1	6	$R_3 - \frac{1}{4}R_1$
P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$

Misread Alternative 3

Increasing *y* first,

b.v.	x	y	z	r	s	t	Value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
s	2	0	2	–1	1	0	4	$R_2 - R_1$
t	–1	0	2	$-\frac{1}{2}$	0	1	10	$R_3 - \frac{1}{2}R_1$
P	–1	0	–2	2	0	0	48	$R_4 + 2R_1$

Increasing *x* next,

b.v.	x	y	z	r	s	t	Value	row ops
y	0	1	2	1	0	0	24	R_1 no change
x	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	0	0	3	–1	$\frac{1}{2}$	1	12	$R_3 - 3R_2$
P	0	0	–1	$\frac{3}{2}$	$\frac{1}{2}$	0	50	$R_4 + R_2$

then increasing *z*.

b.v.	x	y	z	r	s	t	Value	row ops
y	–2	1	0	2	–1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 1$
t	–3	0	0	$\frac{1}{2}$	–1	1	6	$R_3 + R_2$
P	1	0	0	1	1	0	52	$R_4 + R_2$

Misread Alternative 4

Increasing *y* first,

b.v.	x	y	z	r	s	t	Value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
s	2	0	2	–1	1	0	4	$R_2 - R_1$
t	–1	0	2	$-\frac{1}{2}$	0	1	10	$R_3 - \frac{1}{2}R_1$
P	–1	0	–2	2	0	0	48	$R_4 + 2R_1$

then increasing *z* next.

b.v.	x	y	z	r	s	t	Value	row ops
y	–2	1	0	2	–1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	–3	0	0	$\frac{1}{2}$	–1	1	6	$R_3 - 2R_2$
P	1	0	0	1	1	0	52	$R_4 + 2R_2$

(c) P = 52 x = 0 y = 20 z = 2 r = 0 s = 0 t = 6 M1 A1ft A1 3

Notes

1M1: At least 4 values stated. No negative. Reading off bottom row gets M0.
1A1ft: At least 4 values correct.
2A1: cao

[13]

3. (a) $x = 0$, $y = 0$, $z = 2$ B2 1 0 2

Note

1B1: Any 2 out of 3 values correct

2B1: All 3 values correct.

(b) $P - 2x - 4y + \frac{5}{4}r = 10$ M1 A1 2

Note

1M1: One equal sign, modulus of coefficients correct. All the right ingredients.

1A1: cao – condone terms of zero coefficient

[4]

4. (a) e.g. Maximise P = V B1

Subject to: $V - 5p_1 - 3p_2 - 6p_3 + r = 0$ M1

$V - 7p_1 - 8p_2 - 4p_3 + s = 0$ A2,1,0

$V - 2p_1 - 4p_2 - 9p_3 + t = 0$

$p_1 + p_2 + p_3 (+ u) = 1$

where V = value of game to A, P_i = probability of A playing row i

$P_i \geq 0$ and r, s, t, u are stack variables all ≥ 0 B1 5

B1 Maximise/minimise and consistent function
M1 constraints (condone non-negativity)
– at least one correct <u>must be equations</u>
A2 all correct
A1 at least two correct
B1 defining variables

(b) Not reducible and a three variable problem B1 1

B1 cao – both

(c) e.g.

b v	V	P_1	P_2	P_3	r	s	t	u	value		
r	**1**	–5	–3	–6	1	0	0	0	0	M1	
s	1	–7	–8	–4	0	1	0	0	0	A1	
t	1	–2	–4	–9	0	0	1	0	0		2
u	0	1	1	1	0	0	0	1	1		
P	–1	0	0	0	0	0	0	0	0		

b v	V	P_1	P_2	P_3	r	s	t	u	value	Row ops		
V	1	–5	–3	–6	1	0	0	0	0	$R_1 / 1$	M1 A1	
s	0	–2	–5	**2**	–1	1	0	0	0	$R_2 – R_1$	A1	
t	0	–3	–1	–3	–1	0	1	0	0	$R_3 – R_1$	B1ft	
u	0	1	1	1	0	0	0	1	1	R_4 stet		4
P	0	–5	–3	–6	1	0	0	0	0	$R_5 + R_1$		

b v	V	P_1	P_2	P_3	r	s	t	u	value	Row ops	
V	1	–11	–18	0	–2	3	0	0	0	$R_1 + 6R_2$	M1 A1ft
P_3	0	–1	$-\frac{5}{2}$	1	½	½	0	0	0	$R_2 / 2$	A1
t	0	0	$-\frac{17}{2}$	0	$-\frac{5}{2}$	$\frac{5}{2}$	1	0	0	$R_3 + 3R_2$	B1ft
u	0	2	$\frac{7}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	1	$R_4 – R_2$	4
P	0	–11	–18	0	–2	3	0	0	0	$R_5 + 6R_2$	

[16]

1. No Report available for this question.

2. This generated a good spread of marks and differentiated well. The majority of candidates correctly stated the objective function, although some had the signs reversed. Most were able to select the first pivot correctly and went on to produce a good first tableau, with the occasional arithmetic error. The choice of the second pivot caused problems, with many candidates choosing $-\frac{1}{4}$. A negative number may never be selected as a pivot. Many candidates realised they had made an error as negative numbers started to appear for the basic variables and stopped, but others gave negative values for their final answer. Candidates should be reminded that they should state the values of all 7 variables at the end; many only gave the four non-zero values.

3. This question caused problems for most candidates. Very few were able to read off the correct values, or to write down the correct objective function, expressions with two equal signs being the most common error here.

4. Many candidates struggled to answer this question and those that did make a decent attempt encountered difficulties and made a succession of errors. For those candidates who made a reasonable attempt at the question, by far the most popular approach was to divide the probabilities by the value of the game. Unfortunately this then created additional difficulties for the candidate, as for player A it was then necessary to **minimise**, which a large number of candidates failed to state. Candidates also either failed to turn their inequalities into equations or added slack variables when, in this instance, they should have been subtracted. Candidates also failed to define their probabilities etc. Most candidates who tried to answer the question in this way then used the equations for player A in their simplex tableau, not realizing that they needed to change these to player B’s perspective to allow them to **maximise**. Other candidates who started in the same manner, incorrectly set up their equations (inequalities) from B’s perspective, but were then able to use these in their tableau. A minority of candidates adopted one of the other approaches and these candidates were generally more successful. Many candidates failed to mention that simplex was necessary because it was a 3 x 3 problem and it could not be reduced by dominance arguments. Those candidates who reached a correct initial tableau were generally able to manipulate this correctly, although minor errors, either arithmetic or omitting the change of base variable, did occur. Some candidates failed to state the row operations that they had used.